Elastic Behavior of Rigidly Constrained Structural Members

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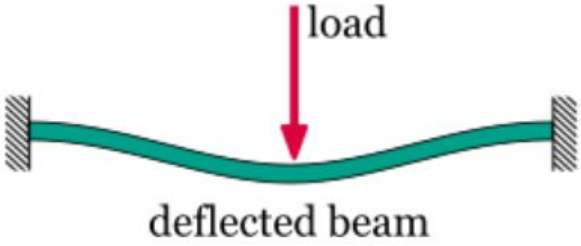


Introduction to Problem

- A fixed-fixed member under load develops bending stress and deflection
- Excessive stress or deflection can cause unsafe performance or failure

Objective

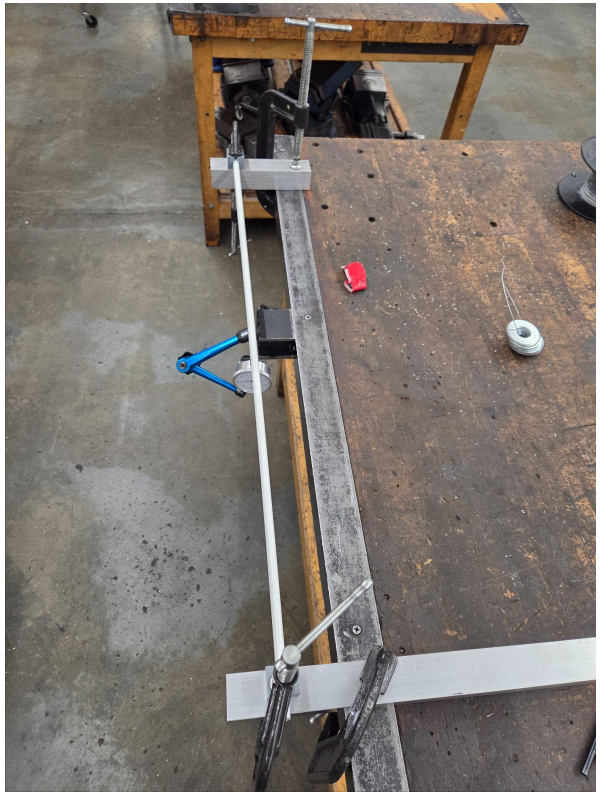
- Calculate maximum stress and deflection to verify safe design

Material

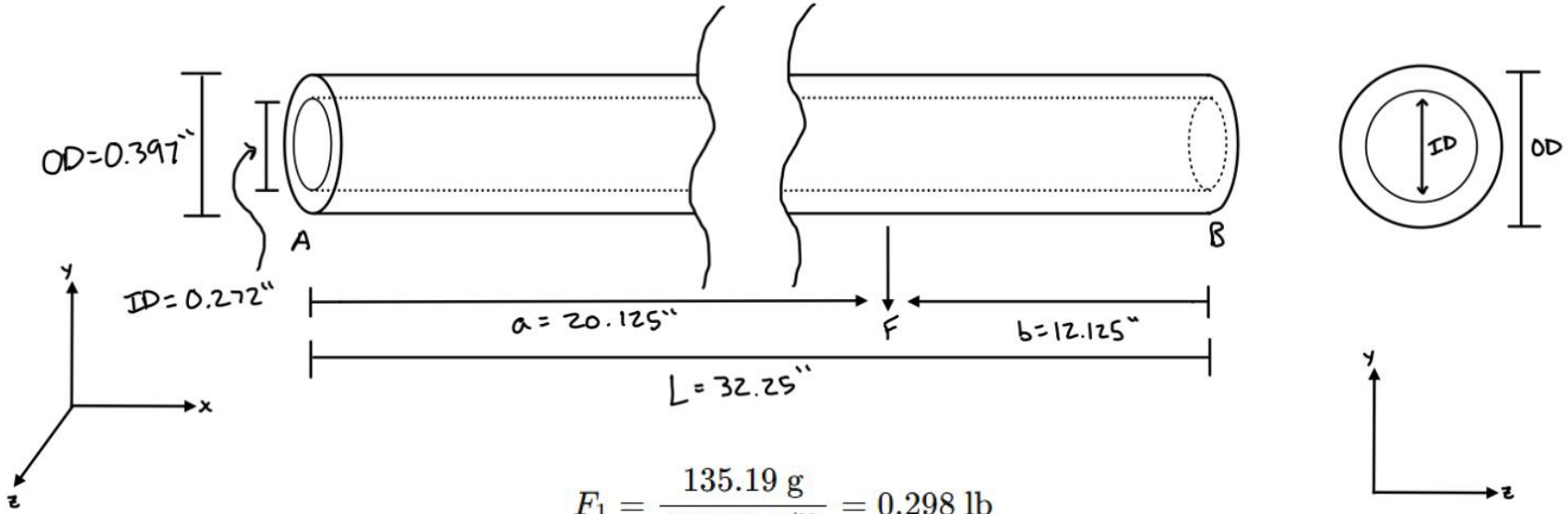
- Material not specified
- Float/Burn test results indicated PVC
- Modulus of elasticity (E) for PVC ranges from 200 - 800 kpsi
- $CaCO_3$ filler used to stiffen material
- Initial assumption E = 400 kpsi
- Later solved for E to match calculations to data

Experiment Setup

Hollow window blinds control wand in bending



Sketch/Figure



Force Conversions:

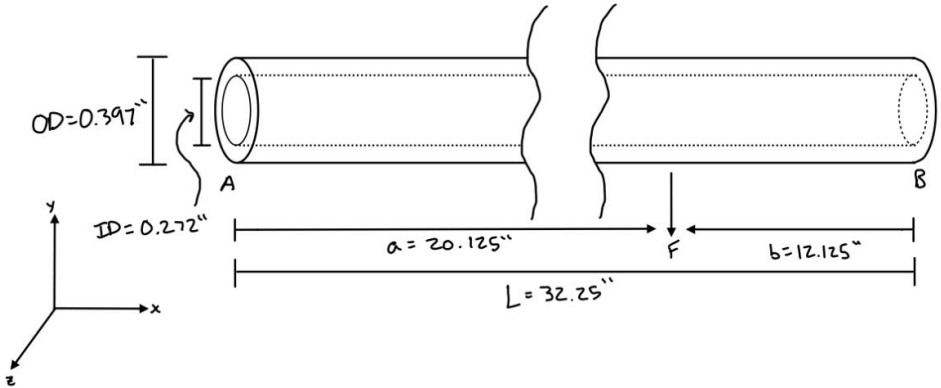
$$F_1 = \frac{135.19\ \text{g}}{453.59\ \text{g/lb}} = 0.298\ \text{lb}$$

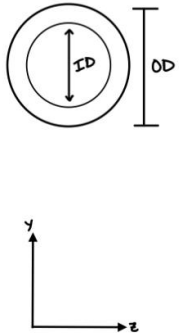
$$F_2 = \frac{251.19\ \text{g}}{453.59\ \text{g/lb}} = 0.553\ \text{lb}$$

$$F_3 = \frac{300.19\ \text{g}}{453.59\ \text{g/lb}} = 0.662\ \text{lb}$$

Rod Properties & Setup

- Calculate Area Moment of Inertia
- This gives bending Stiffness *EI*





$$E = 668{,}000 \text{ psi}$$

$$I = \frac{\pi}{64}(OD^4 - ID^4)$$

$$I = \frac{\pi}{64}(0.397^4 - 0.272^4)$$

$$I = 0.000951 \text{ in}^4$$

Indeterminate Rod Solution

- The rod is statically indeterminate
- Use 0 slope at supports to find the fixed-end moment
- Use equilibrium to find the reaction force

$$M_A = \frac{Fab^2}{L^2}$$

$$M_A = F(2.844)$$

$$R_A = \frac{Fb^2(3a+b)}{L^3}$$

$$R_A = F(0.3177)$$

$$R_B = \frac{Fa^2(a+3b)}{L^3}$$

$$R_B = F(0.6822)$$

Deflection at Midspan

Start with Euler–Bernoulli Beam Theory

$$EI\frac{d^2v}{dx^2} = M(x)$$

For the region left of the load:

$$M(x) = R_A x - M_A$$

After Integrating Twice

$$v(x) = \frac{R_A x^3}{6EI} - \frac{M_A x^2}{2EI}$$

Factor out $\frac{1}{6EI}$:

$$v(x) = \frac{1}{6EI}\left(R_A x^3 - 3M_A x^2\right)$$

Evaluate at Midspan $x = \frac{L}{2}$

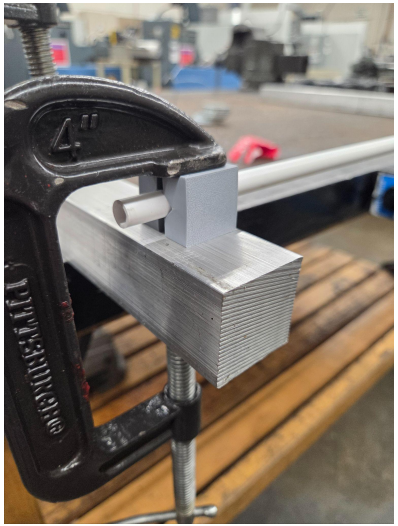
$$\delta\left(\frac{L}{2}\right) = \frac{1}{6EI}\left[R_A\left(\frac{L}{2}\right)^3 - 3M_A\left(\frac{L}{2}\right)^2\right]$$

Final simplified result:

$$\delta = -0.2327\,F$$

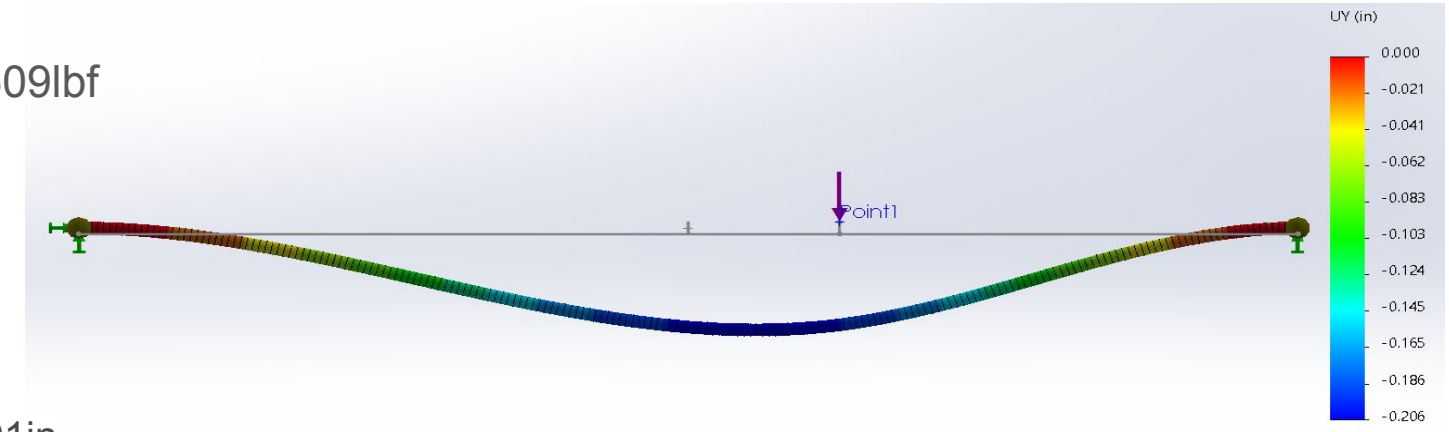
Experimental Procedure

- Both ends fixed using clamps and 3D-printed v-blocks
- Applied mass (steel washers) 4 inches from center
- Measured deflection with zeroed dial indicator for accuracy

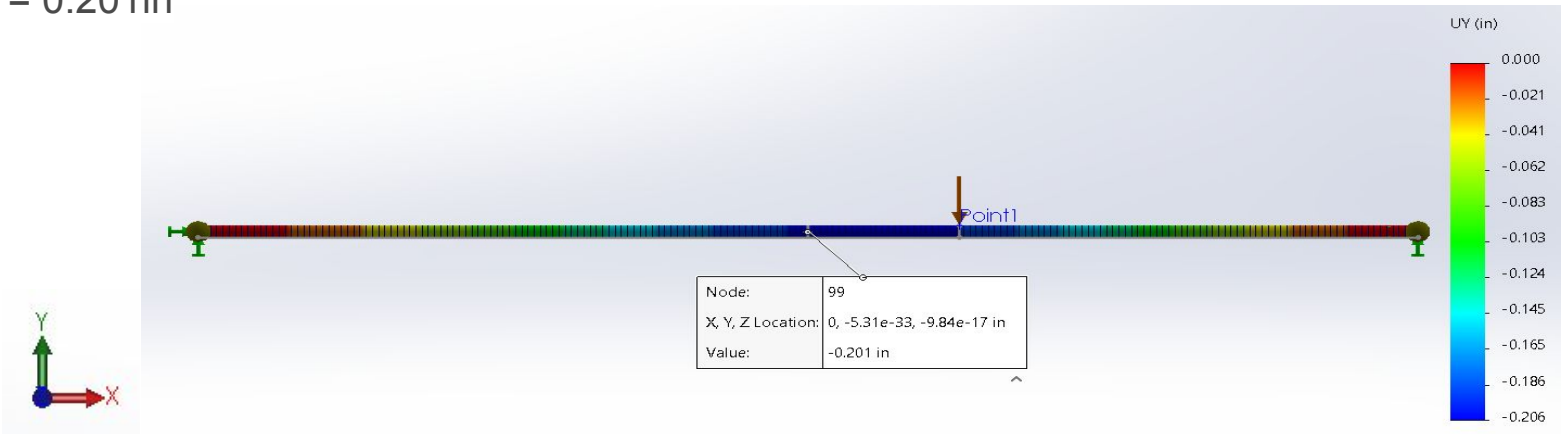


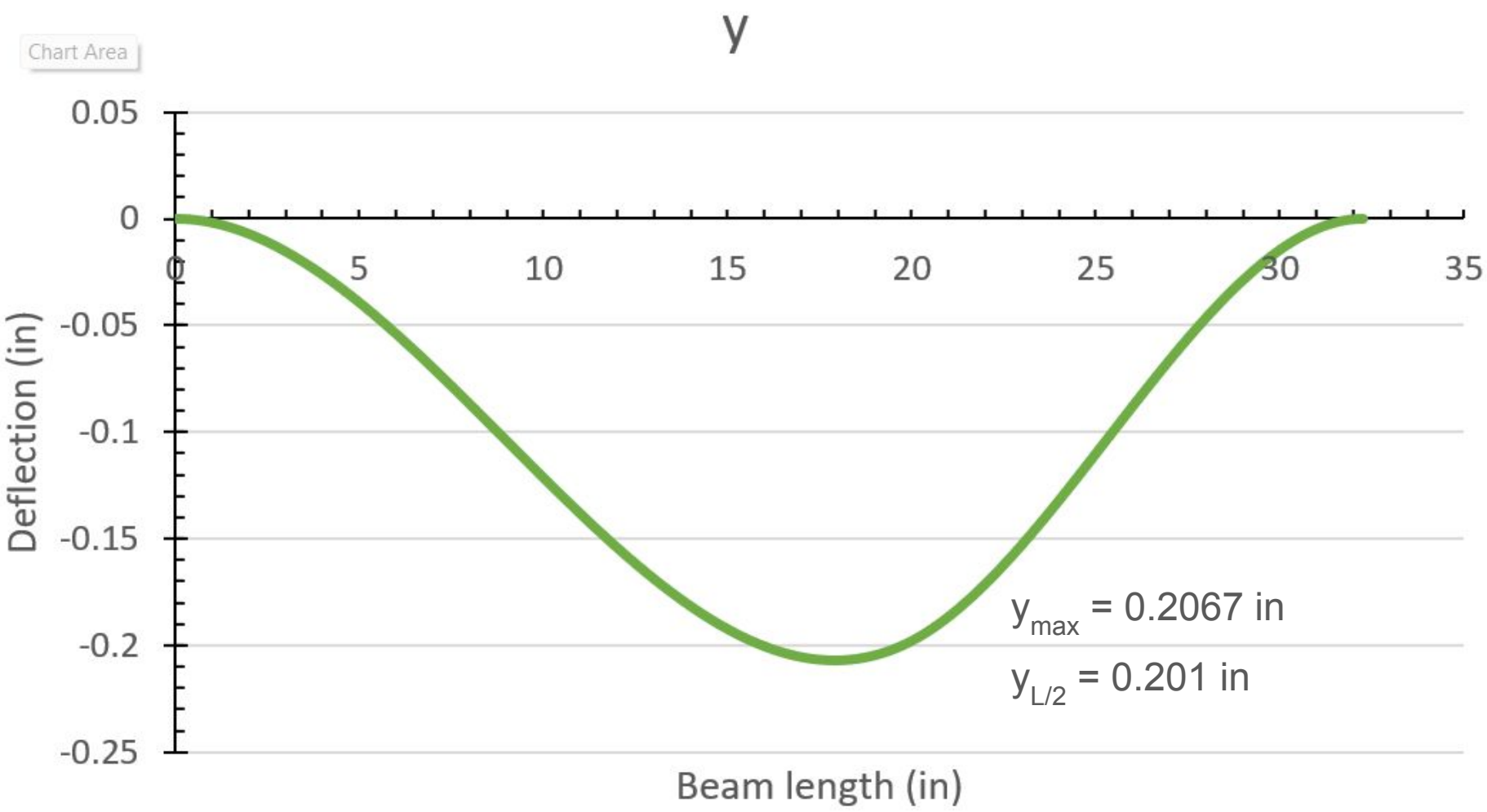
FEA Setup

F = 0.8609lbf



δ = 0.201in




y
Chart Area
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
5
10
15
20
25
30
35
Deflection (in)
Beam length (in)
y_{max} = 0.2067 in
$y_{L/2}$ = 0.201 in

Summary

Method	Test 1 0.2978 lbf	Test 2 0.57289 lbf	Test 3 0.8609 lbf
Experiment	0.064 in	0.145 in	0.201 in
Hand Calculations	0.0693 in	0.1333 in	0.2004 in
Hand Calc Error	8.28%	-8.07%	-0.30%
Singularity Functions	0.0693 in	0.1334 in	0.2004 in
Singularity Error	8.28%	-8%	-0.30%
FEA	0.069 in	0.134 in	0.201 in
FEA Error	7.81%	-7.59%	0.00%

Conclusion

- Error is all below 10%
- E = 668,000psi
- The beam is behaving elastically
- We conclude that the fixed-fixed tube calculations and FEA analysis match the actual behavior of the beam